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Hot electrons under quantization conditions: I. Kinematics

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Abstract. A detailed analysis of the kinematic peculiarities of the interaction of bulk acoustic phonons with confined electrons in a quantum wire is peformed for intra- and inter-sub-band transitions. Due to quantization of the electron motion in a quasi-one-dimensional wire a new parameter $\epsilon_c = (\chi_0 m^* s^2 W_0)^{1/2}$ appears in the kinetic theory (m^* is the electron effective mass, s is the sound velocity, W_0 is the quantum energy of the ground state and χ_0 is some numerical constant which depends upon the shape of the quantizing potential), characterizing the electron–acoustic-phonon interaction. At low lattice temperatures T_0, when $T_0 < \epsilon_c$, the intra-sub-band interaction has a strong inelastic character for the majority of electrons (assuming that the mean electron energy is also less than ε_c). In the opposite case of high lattice temperatures, $T_0 > \epsilon_c$, this interaction is always quasi-elastic for the majority of electrons because the mean electron energy exceeds ε_c. Inter-sub-band scattering, on the other hand, is quasi-elastic at arbitrary lattice temperatures. These kinematic peculiarities of the electron–acoustic-phonon interaction are universal and, in general, do not depend on the physical nature of the quantizing field. It can be an external quantizing magnetic field or size-quantizing electrostatic potential, resulting in the confinement of electrons in a two-dimensional sheet or in a quantum wire, or confinement can be realized due to both electrostatic and magnetic potentials combined. The discussed peculiarities manifest themselves in novel kinetic properties of low-dimensional electron systems.

1. Introduction

A non-equilibrium electron gas in a system of reduced dimensionality such as quasi-one-dimensional (1D) quantum wires (QWIs) provides fertile ground both for fundamental physical studies and for an investigation of possible applications [1]. Electron transport in QWIs can exhibit strongly nonlinear behaviour due to confinement of the motion of particles in two directions, resulting in quantization of the corresponding degrees of freedom. This reveals itself through the quantization of the electron energy spectrum, changes in the density of states of electrons and changes in the probabilities of the electron–acoustic-phonon interaction compared with those of usual three-dimensional (3D) systems. All the above-mentioned characteristics play a crucial role in the electron kinetics of 1D QWIs because the energy spectrum determines the electron group velocity and the scattering probabilities determine the electron momentum and energy dissipation.

Several investigations have now been performed concerning electron transport in QWIs. However, most of them were devoted to the particular situation in which an electron gas is near to the thermodynamic equilibrium state (the linear response approximation). Others were characterized by numerical or Monte Carlo simulation. In spite of the fact that an

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analytical solution provides general insight into the physics which underlies the experimental situation, there have been almost no appropriate studies of non-equilibrium electron kinetics (under the hot-electron approximation) in 1D QWIs based on the Boltzmann equation for the electron distribution function.

The main goal of this paper and the following papers, II and III, is to perform an analytical investigation of hot electrons in 1D QWIs and to obtain new distribution functions for the comprehensive description of the kinetic properties of a 1D electron gas. We consider here the case of a 1D electron gas in a QWI which interacts predominantly with bulk acoustic phonons. The *raison d'être* for this is that the effect of ionized impurity scattering can be virtually eliminated at low temperatures by the physical separation of the impurity atoms from the active QWI channel and, in addition, the electron–electron scattering in a 1D QWI is suppressed due to conservation laws of momentum and energy in a 1D system: electrons simply exchange energies and momenta during collisions. The latter is true in the one-sub-band and two-particle approximations and this property forbids the description of the non-equilibrium distribution of an electron gas in terms of an electron temperature approximation, even if the electron density in a QWI is large. Optical phonon scattering does not affect transport properties of 1D electrons at lattice temperatures below 40 K and electric fields less than a few hundred V cm^{-1} for GaAs QWIs.

Here we propose to study the kinetic problem for a 1D electron gas corresponding to the equivalent problem for a 3D electron gas, namely the classical situation resulting in the so-called Davydov–Druyvestyn distribution function [2]. However, quantization of the motion of electrons along one or two directions in the space leads to some new peculiarities of electron–phonon interaction compared with the 3D case and we have to analyse the physical process in detail. It is noteworthy that these peculiarities are universal and, in general, do not depend on the physical nature of the quantizing field. For example, it can be an external quantizing magnetic field applied to a 3D electron gas, or it can be a size-quantizing electrostatic potential, resulting in the confinement of electrons in a two-dimensional (2D) sheet or in a 1D QWI, or confinement can be realized due to both electrostatic and magnetic potentials combined.

In this paper we describe the fundamental kinematics of the problem, beginning with the analysis of the momentum conservation uncertainty principle in section 2 for the electron–acoustic-phonon interaction. The electronic model is given in section 3. Although we develop a general kinetic theory of the non-equilibrium electrons in a 1D QWI for an arbitrary shape of the quantizing potential, the analytical calculations of the macroscopic characteristics of the 1D electrons in the following papers II and III are performed for a rectangular QWI with an infinitely deep quantum well. We will use this model here to calculate the expression for the form factor. In section 4 the expressions for the scattering rates for non-degenerate statistics are given. The specific kinematic features of the electron–acoustic-phonon interaction in a QWI for intra- and inter-sub-band transitions are discussed in section 5 and summarized in section 6. As mentioned above, the quantum confinement of electrons introduces unusual elements into the kinematics which need to be fully understood before one attempts a description of hot-electron effects. Solution of the Boltzmann equation and its application to non-equilibrium electrons form the subject matters of the following papers II and III.

2. Momentum conservation uncertainty

Historically, the first comprehensive investigation of kinematic peculiarities of the electron–phonon interaction under the quantization of electron motion was published in the review

article of Kubo, Miyake and Hashitsume [3] in connection with the study of galvanomagnetic effects in a 3D electron gas in a strong (quantizing) magnetic field on the basis of Kubo's formula. Later these ideas were used for investigating the power loss in 2D [4] and 1D [5] electron gases and for the calculation of the rate of acoustic-phonon scattering in QWIs [6].

In accordance with [3] the duration of a collision τ_c between an electron and an acoustic phonon cannot be given by the formula $\tau_c \simeq h/\varepsilon_{||}$, where $\varepsilon_{||}$ is the kinetic energy of the electron in the free (non-quantized) direction(s), when $\varepsilon_{||}$ becomes of the same order as or smaller than some characteristic energy $\hbar\omega_{\boldsymbol{q}}^*$ of the acoustic phonon with wavevector $\boldsymbol{q}$. When $\varepsilon_{||}$ is small, the electron velocity in the free direction is small as well, but in such a case the duration τ_c is determined by the motion of the scatterers, namely acoustic-phonons. Under quantization conditions the characteristic extension l^* of the wavepacket of the electron becomes a factor. (In the case of the quantizing magnetic field H this is the classical radius of the ground state Landau orbit, $l^* = \hbar c/(eH)$; in the case of size quantization this is the quantum well width, $l^* = L_\perp$). An acoustic phonon moves with the velocity of sound $\boldsymbol{s}$, so that the duration time τ_c cannot be longer than l^*/s. This is the time during which the acoustic phonon crosses the quantum well where the electrons are localized. The formula $\tau_c \simeq h/\varepsilon_{||}$, cannot be applied when $h/\varepsilon_{||} \geqslant l^*/s$, namely $\varepsilon_{||}^2 \leqslant hs/l^*$. On the other hand, from the quantum picture of the electron–acoustic-phonon interaction it follows that the cross section of the interaction differs from zero only if the phonon wavefunction 'overlaps' the electron wavepacket, namely when the component of the wavevector of the phonon along quantization direction(s) $q_\perp$ is smaller than or of the same order as $q_\perp^* \equiv 2\pi/l^*$,

$$q_\perp \lesssim q_\perp^*. \tag{2.1}$$

The corresponding phonon energy is equal to $\hbar\omega_{\boldsymbol{q}}^* \cong \hbar s q_\perp^* = 2\pi\hbar s/l^*$. Hence, we may expect to reveal some peculiarities of the electron–acoustic-phonon interaction if the kinetic energy of electrons is within the range $\varepsilon_\parallel \lesssim \hbar\omega_{\boldsymbol{q}}^* = 2\pi\hbar s/l^*$. In particular, it is obvious that this interaction will be strongly inelastic, because the electron energy is of the same order as the acoustic phonon energy.

The electron wavefunction which describes the motion of the electrons along the quantization direction(s) represents a packet of plane waves whose characteristic length is l^*. This means that all electron states have an uncertainty for the wavevector along the quantization direction(s) $\Delta\kappa_\perp \simeq \kappa_\perp \simeq 2\pi/l^*$ and there is no precise momentum conservation for the electron–acoustic-phonon interaction in this direction. From the formal point of view, the appearance in the theory of a new parameter $2\pi\hbar/l^*$, with the dimensionality of momentum under the quantization condition, is responsible for new peculiarities of the electron kinetics in such systems compared with 3D ones.

The Boltzmann kinetic equation for the distribution function in the general case is an integro-differential equation. For the electron–acoustic-phonon interaction in the 3D case this equation can be transformed to a pure differential equation [2] due to the existence of a small parameter of quasi-elasticity of the electron–phonon interaction $\delta = \hbar\omega_{\boldsymbol{q}}/\varepsilon(\boldsymbol{\kappa}) \ll 1$, if $\varepsilon(\boldsymbol{\kappa}) > 2m^*s^2$, where $\varepsilon(\boldsymbol{\kappa})$ is the energy of the electron with the wavevector $\boldsymbol{\kappa}$ and m^* is the electron effective mass. If the electron motion undergoes quantization, *a similar parameter does not exist, if the electron kinetic energy is* $\varepsilon_{||} < 2\pi\hbar s/l^*$, as is clear from the previous discussion. As a result we need to solve an integro-differential kinetic equation. The general situation depends on the relationship among appropriate values which include an external electric field E, the lattice temperature T and the parameter $2\pi\hbar s/l^*$. We will show when it is possible to use the quasi-elastic approximation for hot electrons in a 1D QWI and will justify necessary criteria.

3. The electronic model

Here we will develop the kinetic theory of the hot-electron gas in a quantum structure with an arbitrary shape of the size-quantizing potential. However, for the calculation of the kinetic coefficients of the electrons we will deal with a simple confinement configuration [7] arising from, as an example, the elementary GaAs/AlAs potential well which is de-coupled along the two transverse directions, y and z. An electron is confined between infinitely deep potential interfaces at $y = 0$, $y = L_y$ and $z = 0$, $z = L_z$, and is free to move in the x direction of a rectangular QWI. Its normalized wavefunction, which vanishes at the potential interfaces and which has usual periodic properties in the x-direction, is equal to

$$\Psi_{np}(\kappa_x, \boldsymbol{r}) = \frac{2}{V_0^{1/2}} U_{\kappa_x}(x)\, \mathrm{e}^{\mathrm{i}\kappa_x x} \sin(\kappa_y y) \sin(\kappa_x z) \tag{3.1}$$

$$0 \leqslant y \leqslant L_y \qquad 0 \leqslant z < L_z$$

where $\kappa_y = n\pi/L_y, \kappa_z = p\pi/L_z$, n and p are sub-band indices, $V_0 = L_x L_y L_z$ is the volume within which an electron moves, L_x is the QWI length, $U_{\kappa_x}(x)$ is a Bloch periodic function, κ_x is the longitudinal electron wavevector and $\boldsymbol{r} \equiv (x, y, z)$.

The electron energy associated with the state $\Psi_{np}(\kappa_x, \boldsymbol{r})$ is given by

$$\varepsilon_{np}(\kappa_x) = \varepsilon_{||}(\kappa_x) + n^2 W_{0y} + p^2 W_{0z} \tag{3.2}$$

$$\varepsilon_{||}(\kappa_x) = \frac{\hbar\kappa_x^2}{2m^*} \qquad W_{0y,z} = \frac{\pi^2\hbar^2}{2m^* L_{y,z}^2}. \tag{3.3}$$

Here $W_0 = W_{0y} + W_{0z}$ is the quantum energy of the ground electron state ($n = p = 1$).

The electron density of states in the sub-band (n, p) for a given spin and energy $\varepsilon_{np}(\kappa_x) = \varepsilon$ is equal to

$$N_{np}(\varepsilon) = \frac{1}{L_x} \sum_{\kappa_x} \delta(\varepsilon_{np}(\kappa_x) - \varepsilon) = \frac{(2m^*)^{1/2}}{2\pi\hbar} [\varepsilon - (n^2 W_{0y} + p^2 W_{0z})]^{-1/2}. \tag{3.4}$$

The total electron density of states for given spin and energy ϵ is given by the sum

$$N(\varepsilon) = \sum_{np} N_{np}(\varepsilon). \tag{3.5}$$

4. Scattering rates

In the semi-classical limit the rate of electron scattering in a 1D QWI from the initial state (κ_x, ν) to the final state (κ_x', ν') due to the unscreened deformation potential interaction with an acoustic phonon of wavevector $\boldsymbol{q}$ is given by the Fermi golden rule

$$W_{\nu\nu'}^{\pm}(\kappa_x, \kappa_x', \boldsymbol{q}) = \frac{2\pi}{\hbar} |M_{\nu\nu'}^{\pm}(\kappa_x, \kappa_x', \boldsymbol{q})|^2 (N_{\boldsymbol{q}} + \tfrac{1}{2} \pm \tfrac{1}{2}) \delta(\varepsilon_{\nu'}(\kappa_x') - \varepsilon_{\nu}(\kappa_x) \pm \hbar\omega_{\boldsymbol{q}}). \tag{4.1}$$

The upper sign corresponds to emission and the lower sign to absorption of the acoustic phonon (the same everywhere below), $\nu \equiv (n, p)$, $N_{\boldsymbol{q}} = [\exp(\hbar\omega_{\boldsymbol{q}}/T_0) - 1]^{-1}$ is the phonon occupation number (with $T_0 = k_B T$ for brevity), $\omega_{\boldsymbol{q}} = sq$ is the long-wave approximation of the acoustic-phonon dispersion (the linear approximation imposes restriction $q \ll q_B$, where q_B is the maximum wavevector corresponding to the edge of the Brillouin zone) and the square of the electron–phonon matrix element is equal to

$$|M_{\nu\nu'}^{\pm}(\kappa_x, \kappa_x', \boldsymbol{q})|^2 = \frac{\Xi_a^2 \hbar q^2}{2\rho V_0 \omega_{\boldsymbol{q}}} |T_{\nu\nu'}^{\pm}(\kappa_x, \kappa_x', \boldsymbol{q}|^2. \tag{4.2}$$

Here Ξ_a is the deformation acoustic potential, ρ is the density of the matter and the square of the overlap integral (with the cell-periodic part taken to be unity) is given by

$$|T^{\pm}_{\nu\nu'}(\kappa_x, \kappa'_x, \boldsymbol{q})|^2 = G^2_{\nu\nu'}(\boldsymbol{q}_\perp)\delta_{\kappa'_x,\kappa_x\mp q_x} \tag{4.3}$$

where $G^2_{\nu\nu'}(\boldsymbol{q}_\perp)$ is the form factor which takes into account electron quantum confinement in the 1D QWI and depends on the shape of the corresponding electrostatic potential. For the infinite deep rectangular quantum well it is equal to

$$G^2_{\nu\nu'}(\boldsymbol{q}_\perp) = \left| \int_0^{L_y}\int_0^{L_z} \mathrm{e}^{-\mathrm{i}(q_y y+q_z z)}\Phi_{nn'}(y)\Phi_{pp'}(z)\,\mathrm{d}y\,\mathrm{d}z \right|^2 \tag{4.4}$$

where $\boldsymbol{q}_\perp \equiv (q_y, q_z)$ and

$$\Phi_{nn'}(y) = \frac{2}{L_y}\sin\left(\frac{n\pi y}{L_y}\right)\sin\left(\frac{n'\pi y}{L_y}\right) \tag{4.5}$$

$$\Phi_{pp'}(z) = \frac{2}{L_z}\sin\left(\frac{p\pi z}{L_z}\right)\sin\left(\frac{p'\pi z}{L_z}\right). \tag{4.6}$$

The main distinction of the matrix element in equation (4.2) for a 1D QWI from that for the 3D case [2] consists of the replacement of the Kronecker delta function $\delta_{\boldsymbol{\kappa}',\boldsymbol{\kappa}\pm\boldsymbol{q}}$, which reflects momentum conservation in the x, y and z directions for 3D, by the overlap integral from equation (4.3). If translational symmetry exists in the y and z directions as well (the 3D case), then the functions $\Phi(y)$ and $\Phi(z)$ in equations (4.5) and (4.6) have the form

$$\Phi_{3D}(y) = \frac{2}{L_y}\exp[\pm\mathrm{i}(\kappa_y - \kappa'_y)y] \tag{4.7}$$

$$\Phi_{3D}(z) = \frac{2}{L_z}\exp[\pm\mathrm{i}(\kappa_z - \kappa'_z)z] \tag{4.8}$$

and as a result one obtains for the form factor

$$G^2_{3D}(\boldsymbol{q}_\perp) = \delta_{\kappa'_y,\kappa_y\mp q_y}\delta_{\kappa'_z,\kappa_z\mp q_z}. \tag{4.9}$$

We will assume that the 1D QWI is embedded within another material with similar lattice structure and elastic properties and that electrons interact with acoustic phonons which extend over the large cavity of volume V_0 with coupling strengths characteristic of the bulk semiconductor material [8]. That is why the electron–acoustic-phonon matrix element in equation (4.2) for a 1D QWI is exactly transformed to the one for a 3D semiconductor [9]

$$|M^{\pm}_{3D}(\boldsymbol{\kappa}, \boldsymbol{\kappa}', \boldsymbol{q})|^2 = \frac{\Xi_a^2\hbar q^2}{2\rho V_0\omega_{\boldsymbol{q}}}\delta_{\boldsymbol{\kappa}',\boldsymbol{\kappa}\mp\boldsymbol{q}} \tag{4.10}$$

if we reconstruct the translation symmetry in the transverse directions y and z, namely when equation (4.9) applies.

Calculation of the form factor for a rectangular 1D QWI results in the following expressions [8]:

$$G^2_{\nu\nu'}(\boldsymbol{q}_\perp) = G^2_{nn'}(q_y)G^2_{pp'}(q_z) \tag{4.11}$$

$$G^2_{rr'}(q_\alpha) = (\pi^2 rr')^2\frac{(q_\alpha L_\alpha/2)^2\sin^2(q_\alpha L_\alpha/2 + (\pi/2)(r+r'))}{[(q_\alpha L_\alpha/2)^2 - (\pi^2/4)(r-r')^2]^2[(q_\alpha L_\alpha/2)^2 - (\pi^2/4)(r+r')^2]^2} \tag{4.12}$$

where $r = n$, p and $\alpha = y, z$.

This equation shows us that there is some characteristic value of the transverse component of the phonon wavevector approximately equal to

$$q_{\alpha 0} = 2\pi/L_\alpha. \tag{4.13}$$

The form factor $G^2_{rr'}(q_\alpha)$ goes to zero if $q_\alpha > q_{\alpha 0}$, as is clear from figure 1. This means that electrons in a 1D QWI interact with those acoustic phonons which have a transverse component restricted by the condition

$$q_\alpha \lesssim q_{\alpha 0}. \tag{4.14}$$

It is obvious that the form factor of equation (4.12) for a 1D QWI as distinct from the Kronecker delta function from equation (4.9) for the 3D case expresses a profound modification of the electron–acoustic-phonon interaction for quasi-1D electrons.

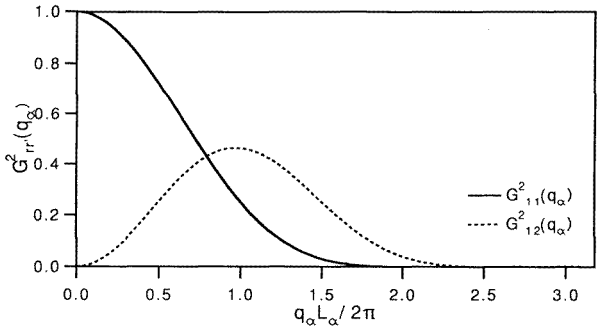

Figure 1. The form factor $G^2_{rr'}(q_\alpha)$ as a function of the normalized transverse component of the acoustic-phonon wavevector $q_\alpha L_\alpha/(2\pi)$ $(\alpha = y, z)$ for intra-sub-band $(r = r' = 1)$ and inter-sub-band $(r = 1, r' = 2)$ transitions.

5. Kinematics of the electron–phonon interaction

As mentioned above, the quantum confinement of electrons introduces unusual elements into the kinematics which need to be fully understood before one attempts a kinetic description of the non-equilibrium electron effects. Some important peculiarities of the electron–acoustic-phonon interactions in a 1D QWI can be revealed from the analysis of energy and longitudinal momentum conservation which are described by the argument of the delta-function of equation (4.1):

$$\varepsilon_{\nu'}(\kappa_x \mp q_x) - \varepsilon_\nu(\kappa_x) \pm \hbar\omega_{\boldsymbol{q}} = 0. \tag{5.1}$$

A similar analysis for a 3D electron gas in a quantizing magnetic field was performed in [10] and here we will follow this method.

By using equations (3.2) and (3.3) we obtain the following equation for the q_x and $q_\perp$ components of the wavevector $\boldsymbol{q}$ of the acoustic phonon which interacts with the electron of wave-vector κ_x in sub-band ν:

$$q_x^2 \mp 2\kappa_x q_x \pm \frac{2m^* s}{\hbar}(q_x^2 + q_\perp^2)^{1/2} - \frac{2m^* s}{\hbar}(W_\nu - W_{\nu'}) = 0 \tag{5.2}$$

where $W_\nu = n^2 W_{0y} + p^2 W_{0z}$.

The character of the electron–acoustic-phonon interaction is quite different for intra-sub-band $(\nu = \nu')$ and inter-sub-band $(\nu \neq \nu')$ transitions.

5.1. Intra-sub-band scattering

For intra-sub-band scattering we have the equation

$$q_x^2 \mp 2\kappa_x q_x \pm \frac{2m^* s}{\hbar}(q_x^2 + q_\perp^2)^{1/2} = 0 \tag{5.3}$$

whose solution is given by

$$q_\perp = \frac{\hbar|q_x|}{2m^* s}\left[\left(q_x \mp 2\kappa_x + \frac{2m^* s}{\hbar}\right)\left(q_x \mp 2\kappa_x - \frac{2m^* s}{\hbar}\right)\right]^{1/2}. \tag{5.4}$$

From equation (5.4) we have the following relationship between $q_\perp$ and $|q_x|$ (see also [3, 10] for a 3D electron gas in a quantizing magnetic field):

$$q_\perp \gg |q_x| \tag{5.5}$$

for all possible values of q_x excluding a small region (of the order of a few units of $2m^* s/\hbar$ near to the points $q_x = \pm 2\kappa_x$). For GaAs parameters ($m^* = 0.07m_0$, $s = 5.14\times 10^5$ cm s^{-1}, m_0 is the mass of a free electron) $2m^* s/\hbar = 6.2 \times 10^4$ cm^{-1} and then $2\kappa_x \gg 2m^* s/\hbar$ for electron kinetic energies higher than 10^{-2} meV. As a consequence we can neglect $2m^* s/\hbar$ inside the square brackets in equation (5.4) or, which amounts to the same thing, neglect q_x^2 compared with $q_\perp^2$ in the third term in equation (5.3). Then equation (5.4) is transformed into the very simple form

$$q_\perp = \mp\frac{\hbar}{2m^* s}(q_x^2 \mp 2\kappa_x q_x) = \mp\frac{\varepsilon_\parallel(\kappa_x)}{\hbar s}\left[\left(\frac{q_x}{\kappa_x}\right)^2 \mp 2\left(\frac{q_x}{\kappa_x}\right)\right]. \tag{5.6}$$

It is of interest to compare equation (5.6) for a 1D electron gas with the analogous one for a 3D electron gas ($|\kappa| = \kappa_x$) [11]. For the same approximation as above we obtain the equation

$$(q_x \mp \kappa)^2 + q_\perp^2 = \kappa^2 \tag{5.7}$$

or, in more suitable form for comparison with equation (5.6):

$$q_\perp = (-q_x^2 \pm 2\kappa q_x)^{1/2}. \tag{5.8}$$

As we can see, the remarkable peculiarity of equation (5.6) is the presence of the small parameter $2m^* s \ll \hbar\kappa_x$ in the denominator of this equation, resulting in the inequality in equation (5.5). This does not happen in equation (5.8). Figures 2(a) and (b) demonstrate schematically the interdependence of the components q_x and $q_\perp$ for 1D and 3D electron gases. Here we show separately the branches for forwards and backwards scattering. For 1D electrons the scattering is backwards or forwards depending on whether the electron wavevector direction is reversed or not during the scattering event. For 3D electrons we call the scattering backwards if the projection of the final state electron wavevector $\boldsymbol{\kappa}' = \boldsymbol{\kappa} \mp \boldsymbol{q}$ onto the direction of the initial state wavevector $\boldsymbol{\kappa}$ is anti-parallel to $\boldsymbol{\kappa}$ and we call the scattering forwards if this projection is parallel to $\boldsymbol{\kappa}$. It is evident from figures 2(*a*) and (*b*) that, for the 3D electron gas, we have the relationship

$$q_\perp \simeq |q_x| \simeq \kappa \tag{5.9}$$

which is distinctly different from equation (5.5) for a 1D QWI.

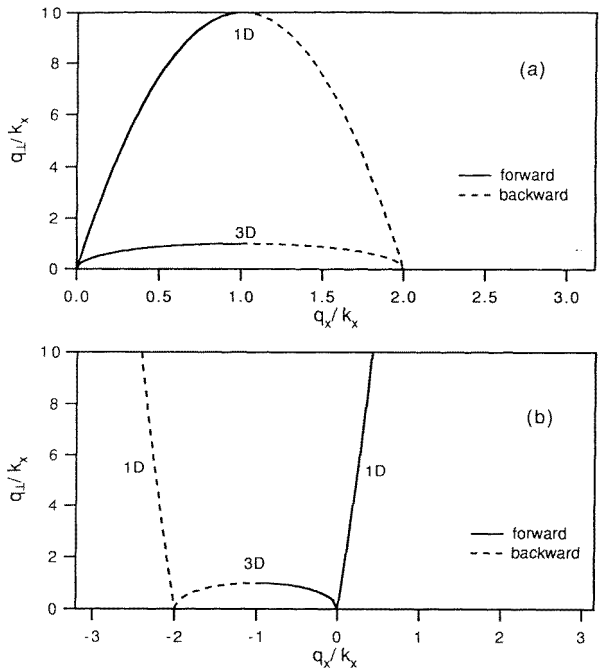

Figure 2. A schematic representation of the interdependence between longitudinal q_x and transverse $q_\perp$ components of the acoustic-phonon wavevector $\boldsymbol{q}$ for interaction with electrons in a 1D QWI (intra-sub-band scattering) and in a 3D semiconductor: (a) for the emission process and (b) for the absorption process. It is assumed that $\kappa_x > 0$ and $\overline{h}\kappa_x/(2m^*s) = 10$ for the 1D case, whereas $|\boldsymbol{\kappa}| = \kappa_x$ for the 3D case. The full line plots forwards scattering; the broken line plots backwards scattering.

5.1.1. Spontaneous emission. From the analysis of the kinematics of the intra-sub-band electron–phonon interaction in a 1D QWI it follows that, *in principle*, electrons can emit acoustic phonons predominantly with wavevectors almost normal to the axis of the 1D QWI in accordance with equation (5.5). However, any realization of this possibility for the spontaneous emission process depends on the particular shape of the $\boldsymbol{q}$-dependence of the rate of electron scattering in equation (4.1). For the deformation acoustic potential the square of the matrix element in equation (4.2) is proportional to q, $|M^{\pm}_{\nu\nu'}(\kappa_x, \kappa'_x, \boldsymbol{q})|^2 \simeq q$, and as a result electrons interact more strongly with phonons with large q, for which

$$q = (q_\perp^2 + q_x^2)^{1/2} \simeq q_\perp \tag{5.10}$$

in accordance with equation (5.5) and figure 1. For the piezoelectric acoustic potential we have [9] for $|M_{\nu\nu'}(\kappa_x, \kappa'_x, \boldsymbol{q})|^2 \simeq 1/q$. This means that electrons interact more strongly with phonons with small q; that is, the important regions are near to points $(q_x = 0, q_\perp = 0)$ (for forwards scattering) and $(q_x = \pm 2\kappa_x, q_\perp = 0)$ (for backwards scattering), as is obvious from figure 2. In the last case we cannot use equation (5.5). (This conclusion would have to be revised in the presence of screening.)

Because we restrict our analysis here to the deformation interaction we will exploit equations (5.5) and (5.10). In the emission process the energy of the emitted acoustic

phonon from equation (5.6) is

$$\hbar\omega_{\boldsymbol{q}} = \hbar s q \simeq \hbar s q_{\perp} \simeq \hbar s \frac{\varepsilon_{\parallel}(\kappa_x)}{\hbar s} = \varepsilon_{\parallel}(\kappa_x). \tag{5.11}$$

Consequently, the intra-sub-band interaction between electrons and acoustic phonons is essentially inelastic in a 1D QWI (see also [3, 6, 10]), in contrast to the situation in a 3D electron gas [11]. It should be particularly emphasized that this inference about inelasticity of scattering is evident from the analysis of conservations laws in equation (5.1), taking into account the shape of the $\boldsymbol{q}$-dependence of the matrix element in equation (4.2). No assumptions concerning the shape of the form factor in equation (4.3) have been made.

To justify when equations (5.5), (5.10) and (5.11) hold it is necessary to take into account additional conditions deriving from the form factor. The maximum value of the transverse component is equal to

$$q_{\perp 0} = (q_{y0}^2 + q_{z0}^2)^{1/2} \equiv \left(\frac{8m^* W_0}{\hbar^2}\right)^{1/2}. \tag{5.12}$$

Then for the spontaneous emission processes the form factor will not affect the electron–acoustic-phonon interaction if $q_{\perp 0} > \varepsilon_{\parallel}(\kappa_x)/(\hbar s)$, or, in terms of electron kinetic energy

$$\varepsilon_{\parallel}(k_x) < \hbar s q_{\perp 0} = (8m^* s^2 W_0)^{1/2}. \tag{5.13}$$

Because $q_{\perp} < \varepsilon_{\parallel}(\kappa_x)/(\hbar s) < q_{\perp 0}$ we can put in equation (4.12) for the form factor

$$G_{\nu\nu}^2(\boldsymbol{q}_{\perp}) \simeq 1 \tag{5.14}$$

for the spontaneous emission processes. Restrictions on q_x and $q_{\perp}$ for this case are given only by the kinematics of the electron–phonon interactions, that is by equation (5.1). Equation (5.13) thus defines the range of the electron energy within which the interaction with acoustic phonons is inelastic.

On the other hand, if the electron energy is in the range

$$\varepsilon_{\parallel}(\kappa_x) > (8m^* s^2 W_0)^{1/2} \tag{5.15}$$

then the form factor will restrict the maximum value of $q_{\perp}$, $(G_{\nu\nu}^2(q_{\perp}) \neq 1)$, and, in accordance with equation (4.14) and previous analysis, we can put $q_{\perp} \simeq q_{\perp 0}$ in equation (5.3) for the emission processes:

$$q_x^2 - 2\kappa_x q_x + \frac{2m^* s}{\hbar}(q_x^2 + q_{\perp 0}^2)^{1/2} = 0. \tag{5.16}$$

If $q_{\perp 0} > |q_x|$ we have the solution

$$q_x = \kappa_x \pm \left(\frac{2m^*}{\hbar^2}\right)^{1/2} [\varepsilon_{\parallel}(\kappa_x) - (8m^* s^2 W_0)^{1/2}]^{1/2} \tag{5.17}$$

where the second term in the square brackets is relatively small, in accordance with equation (5.15). If $q_{\perp 0} < |q_x|$ then we have solutions

$$q_x = 0 \qquad q_x = 2\kappa_x \pm \frac{2m^* s}{\hbar} \tag{5.18}$$

where the second term is small compared with $2\kappa_x$. This means that, for arbitrary q_x, we can neglect the last term in equation (5.1). We conclude that the electron–phonon interaction for the spontaneous emission processes is inelastic if the electron kinetic energy is defined by equation (5.13) and quasi-elastic if it is defined by equation (5.15).

5.1.2. Absorption and stimulated emission. The analysis above is adequate for spontaneous emission but for the absorption and stimulated emission processes there is an additional restriction on q. This is due to the proportionality of the corresponding scattering rate in equation (4.1) to the phonon occupation number N_q, which, to be other than negligible, entails the condition

$$(q_x^2 + q_\perp^2)^{1/2} \lesssim \frac{T_0}{\hbar s}. \tag{5.19}$$

Note that, for a large $q (q > T_0/(\hbar s))$, N_q has an exponentially small value. The character of these processes depends on the relationship between $q_{\perp 0}$ and $T_0/(\hbar s)$. If $q_{\perp 0} < T_0/(\hbar s)$, or

$$T_0 > (8m^* s^2 W_0)^{1/2} \tag{5.20}$$

then the restriction embodied in equation (4.14) (namely that $q_\perp \lesssim q_{\perp 0}$) is more severe than is the one from equation (5.19). Hence, the electron–phonon interaction for the absorption and stimulated emission processes has exactly the same peculiarities as for the spontaneous emission processes which were described above. Only the additional large factor $N_q \approx T_0/(\hbar s q)$ appears in equation (4.1) for the processes discussed compared with the spontaneous emission process.

On the other hand, if $q_{\perp 0} > T_0/(\hbar s)$, or

$$T_0 < (8m^* s^2 W_0)^{1/2} \tag{5.21}$$

then the restriction equation (5.19) is more severe than equation (4.14). Because in this case $q \lesssim T_0/(\hbar s) < q_{\perp 0}$, we can use the approximation of equation (5.14) for the form factor for arbitrary electron kinetic energy and, as a result, all peculiarities of electron–phonon interaction for the absorption and stimulated emission processes are determined only by the energy and momentum conservation, namely by equation (5.3). (Of course, the factor N_q has to be taken into account in equation (4.1)). A similar analysis to that performed for spontaneous emission processes shows us that the last term in this equation is important if

$$\varepsilon_\parallel(\kappa_x) < T_0 \tag{5.22}$$

and unimportant if

$$\varepsilon_\parallel(\kappa_x) > T_0. \tag{5.23}$$

This implies that, for the lattice temperatures defined by equation (5.21), the electron–acoustic-phonon interaction for absorption and stimulated emission processes is inelastic for the electron kinetic energy defined by equation (5.22) and it is quasi-elastic for the energy defined by equation (5.23), an implication that is physically clear if we remember that (in accordance with equation (5.19)) the maximum value of the absorbed phonon energy is $\hbar\omega_q \lesssim T_0$. Figure 3 shows the corresponding lattice temperature and the electron energy regions with inelastic and quasi-elastic scattering for the spontaneous emission processes and stimulated emission and absorption processes.

To finish this kinematic analysis of the intra-sub-band electron–phonon scattering let us define the range of the electron energy within which the relationship equation (5.5) ($q_\perp \gg |q_x|$) between $q_\perp$ and q_x is satisfied. It is obvious from equation (5.6) that equation (5.5) is realized automatically within the energy range in equation (5.13), but the real energy range within which equation (5.5) holds is wider than this. In fact, for an electron energy defined by equation (5.15), within which the electron–phonon interaction is quasi-elastic, we have for the spontaneous emission process from equation (5.3) $|q_x| \simeq 2|\kappa_x|$ and from equation (4.14) $q_\perp \simeq q_{\perp 0}$. Then $q_\perp > |q_x|$ for a kinetic energy given by

$$\varepsilon_\parallel(\kappa_x) < W_0. \tag{5.24}$$

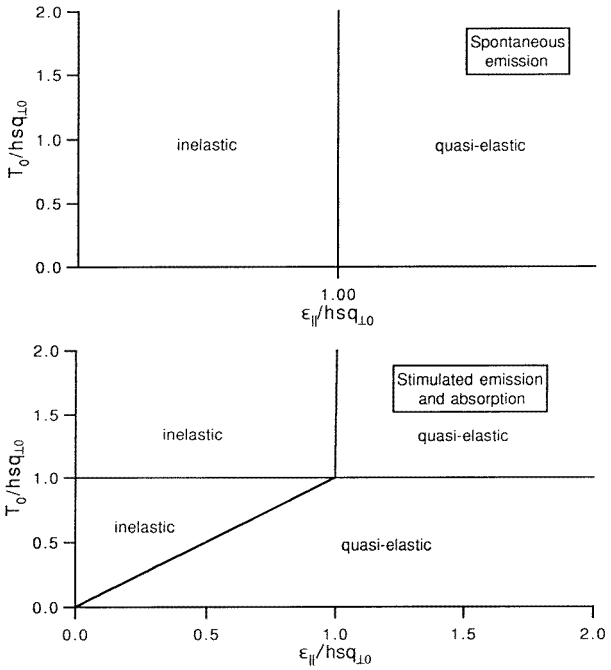

Figure 3. Regions of the lattice temperatures T_0 and the electron energies $\varepsilon_{\parallel}(\kappa_x)$ with inelastic and quasi-elastic electron–acoustic-phonon interaction.

The same is true for the absorption and stimulated emission processes if the lattice temperature is defined by equation (5.20). Electrons with kinetic energy in the range defined by equation (5.24) will predominantly emit and absorb acoustic phonons with wavevectors almost perpendicular to the QWI axis.

For the lattice temperature defined by equation (5.21) we will have $q_{\perp} > |q_x|$ for the absorption and stimulated emission processes if the electron energy is in the range

$$\varepsilon_{\parallel}(k_x) < \frac{T_0^2}{8m^*s^2}. \tag{5.25}$$

This is true because, for an electron energy defined by equation (5.22), the inequality $q_{\perp} > |q_x|$ is automatically realized due to equation (5.6), whereas for the energy range from equation (5.23) we have $|q_x| \simeq 2|\kappa_x|$ and $q_{\perp} \simeq T_0/(\hbar s)$, resulting in equation (5.25).

For the electron energies which are outside the regions defined in equations (5.24) and (5.25), we have the opposite relationship between $q_{\perp}$ and $|q_x|$, namely $q_{\perp} < |q_x|$. Electrons will predominantly interact with acoustic phonons with wavevectors along the QWI axis. Note that this is the case when the momentum conservation approximation (MCA) [8] can be used with high accuracy.

Note that the energy region defined by equation (5.13) exists for intra-sub-band scattering within every sub-band and that $\varepsilon_{\parallel}(\kappa_x)$ is the electron kinetic energy calculated from the bottom of the corresponding sub-band. For upper sub-bands this region will be slightly increased compared with the first sub-band due to the increased magnitude of $q_{\perp 0}$ (as follows from equation (4.12) and figure 2). The same is true for equations (5.24) and (5.25).

5.2. Inter-sub-band scattering

For inter-sub-band ($\nu \neq \nu'$) scattering we can neglect the phonon energy $\hbar\omega_q$ in equation (5.1) for all transitions. In fact, because the minimum value for the difference $|W_\nu - W_{\nu'}|$ in equation (5.2) is $|W_\nu - W_{\nu'}|_{min} > W_0$ and

$$\frac{2m^*}{\hbar^2}|W_\nu - W_{\nu'}|_{min} > \frac{q_{\perp 0}^2}{4}$$

then we can neglect $(2m^*/\hbar)(q_x^2+q_\perp^2)^{1/2}$ compared either with the second or with the fourth term in equation (5.2):
if $q_\perp > q_x$ then

$$\frac{2m^*s}{\hbar}(q_x^2+q_\perp^2)^{1/2} \simeq \frac{2m^*s}{\hbar}q_\perp \ll \frac{q_{\perp 0}^2}{4}$$

if $q_x > q_\perp$ then

$$\frac{2m^*s}{\hbar}(q_x^2+q_\perp^2)^{1/2} \simeq \frac{2m^*s}{\hbar}|q_x| \ll 2|\kappa_x q_x|.$$

As a result, inter-sub-band scattering in a 1D QWI is *always quasi-elastic*; that is, $\hbar\omega_q \ll [\varepsilon_{\nu'}(\kappa_x \mp q_x), \varepsilon_\nu(\kappa_x), |W_\nu - W_{\nu'}|]$ (see a similar result for a 3D electron gas in a quantizing magnetic field in [10]). Thus the statement in [6] that the inter-sub-band transitions due to acoustic-phonon scattering are even more inelastic (compared with intra-sub-band scattering) is not true.

Equation (5.2) is transformed to

$$q_x^2 \mp 2\kappa_x q_x - \frac{2m^*}{\hbar^2}(W_\nu - W_{\nu'}) = 0 \tag{5.26}$$

and its solutions are given by

$$q_x = \kappa_x \pm \left(\kappa_x^2 + \frac{2m^*}{\hbar^2}(W_\nu - W_{\nu'})\right)^{1/2} \tag{5.27}$$

for emission and

$$q_x = -\kappa_x \pm \left(\kappa_x^2 + \frac{2m^*}{\hbar^2}(W_\nu - W_{\nu'})\right)^{1/2} \tag{5.28}$$

for absorption. Recall that the electron wavevector κ_x corresponds to the kinetic energy in the sub-band with index ν.

For emission processes it follows from equation (5.27) that, when $\nu > \nu'$, we have $|q_x| \simeq q_{\perp 0}$ for backwards and forwards scattering if $\varepsilon_\parallel(\kappa_x) \ll (W_\nu - W_{\nu'})$. Alternatively, if $\varepsilon_\parallel(\kappa_x) \gg (W_\nu - W_{\nu'})$, we have $|q_x| \simeq 2|\kappa_x|$ for backwards and $|q_x|^2 \simeq q_{\perp 0}^2/|\kappa_x|$ for forwards scattering. Since $q_\perp \simeq q_{\perp 0}$ it means that, in the first case, electrons will emit acoustic phonons with an isotropic distribution ($|q_x| \simeq q_\perp \simeq q_{\perp 0}$), whereas, in the second case, the phonon wavevector is along the QWI axis ($q \simeq |q_x| \simeq 2|\kappa_x| > q_\perp$) for backwards scattering and normal to the QWI axis ($q \simeq q_\perp \simeq q_{\perp 0} > |q_x|$) for forwards scattering. For inter-sub-band transitions with $\nu < \nu'$ the same is true for the energy ranges $\varepsilon_\parallel(\kappa_x) \gtrsim |W_\nu - W'_\nu|$ and $\varepsilon_\parallel(\kappa_x) \gg |W_\nu - W_{\nu'}|$, respectively.

A similar picture of the inter-sub-band transition is valid for the absorption processes in accordance with equation (5.28) for high lattice temperatures, defined by equation (5.20). For low lattice temperatures, from equation (5.21) the inter-sub-band transitions are suppressed due to the small exponential factor $N_q \simeq \exp(-\hbar s q_{\perp 0}/T_0)$.

Clearly, the peculiarities of the electron–acoustic-phonon interaction in a 1D QWI previously described will bring about a radical modification of the hot-electron kinetics compared with that in a 3D electron gas.

6. Summary

In this paper we have described the basic kinematics of the interaction of bulk acoustic phonons with confined electrons in a quantum wire. Althoguh we have specialized the discussion to the interaction via a deformation potential, much is applicable to piezoelectric interactions. Our assumption of strong confinement for the electrons allowed us to make a sharp distinction between $q_\perp$ and q_x, the phonon wavevector components perpendicular to and along the QWI, respectively, and to introduce a new characteristic energy $\varepsilon_c = (8m^*s^2W_0)^{1/2}$. For a GaAs rectangular QWI with $L_y = L_z \equiv L_\perp = 50$ Å, $m^* = 0.07m_e$ and $s = 5.14 \times 10^5$ cm s^{-1}, we obtain $W_0 = \pi^2\hbar^2/(m^*L_\perp^2) = 430$ meV, $m^*s = 0.01$ meV and $\varepsilon_c = 6$ meV $\simeq 70$ K, so ε_c is typically of comparable magnitude with electron energies. Whereas q_x is restricted by momentum conservation along the axis, $q_\perp$ is restricted by energy conservation and by the form factor to be below $q_{\perp 0}$. Our characteristic energy is just $\varepsilon_c = \hbar s q_{\perp 0}$ and it follows that, if $\varepsilon_\parallel < \varepsilon_c$, the processes involving spontaneous emission are inelastic, whereas if $\varepsilon_\parallel > \varepsilon_c$ they are quasi-elastic. This will be true also of processes involving absorption and stimulated emission provided that equipartition holds, that is $T_0 > \hbar s q_{\perp 0}$. If equipartition does not hold, the maximum phonon energy is T_0 for the stimulated processes and so the conditions for inelastic or quasi-elastic scattering become $\varepsilon_\parallel < T_0$ and $\varepsilon_\parallel > T_0$, respectively. All these conditions for intra-sub-band scattering are summarized in figure 3. For inter-sub-band scattering the phonon energy is always much smaller than the electron energies involved and so all such processes as quasi-elastic.

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